Unit 2

Synthesis of plane mechanism

KINEMATICS FUNDAMENTALS

- ➤ Degrees of Freedom
- ➤ Types of Motion
- ➤ Links, Joints, and Kinematic chains
- ➤ Determining Degree of Freedom
  - ➤ Degree of Freedom in Planar Mechanisms
- ➤ Mechanisms and Structures
- ➤ Number Synthesis
- ⊙ Dimensional synthesis
- ⊙ Motion generation
- ⊙ Path generation.
- ⊙ Function generation
- ⊙ Complex number
- ⊙ modeling graphical
- ⊙ Analytical and combined methods of synthesis

DEGREES OF FREEDOM

- The system's DOF is equal to *the number of independent parameters (measurements) which are needed to uniquely define its position in space at any instant of time.*

- *A rigid body in plane motion has three DOF.*

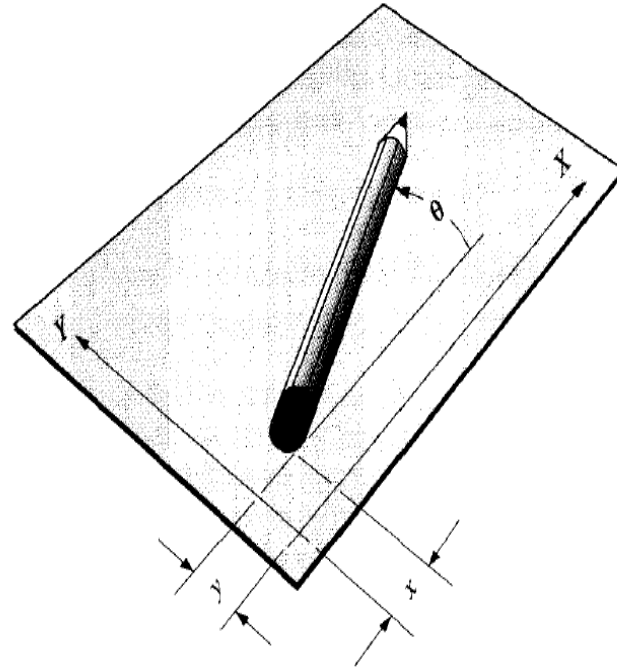- *Any rigid body in three-space has six degrees of freedom*

FIGURE 2-1

A rigid body in a plane has three *DOF*

3

TYPES OF MOTION

- A rigid body free to move within a reference frame will, in the general case, have **complex motion**, which is a simultaneous combination of **rotation** and **translation**.

- In three-dimensional space, there may be rotation about any axis and also simultaneous translation which can be resolved into components along three axes. In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane.

- we will limit our present discussions to the case of **planar (2-D) kinematic systems.**

TYPES OF MOTION

- **Pure rotation**

  the body possesses one point (center of rotation) which has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

- **Pure translation**

  all points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation.

- **Complex motion**

  a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

LINKS, JOINTS, AND KINEMATIC CHAINS

- A **link** is an (assumed) rigid body which possesses at least two **nodes** which are *points for attachment to other links.*

Nodes

Binary link   Ternary link   Quaternary link
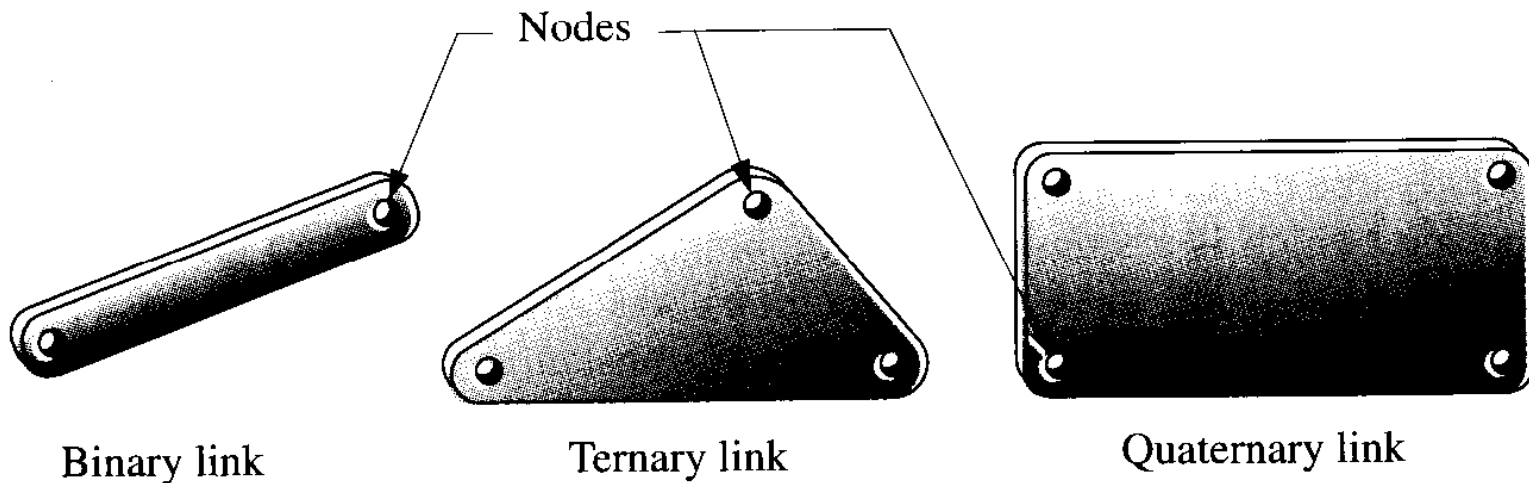
LINKS, JOINTS, AND KINEMATIC CHAINS

- Some of the common types of links are:

- **Binary link** - *one with two nodes.*
- **Ternary link** - *one with three nodes.*
- **Quaternary link** - *one with four nodes.*
- ***Pentagonals*** *– one with five nodes.*
- ***Hexagonals*** *– one with six nodes*

LINKS, JOINTS, AND KINEMATIC CHAINS

- A **joint** is *a connection between two or more links (at their nodes), which allows some motion, or potential motion, between the connected links. Joints (also called **kinematic pairs**) can be* classified in several ways:

1. By the type of contact between the elements, line, point, or surface.

2. By the number of degrees of freedom allowed at the joint.

3. By the type of physical closure of the joint: either **force** or **form** closed.

4. By the number of links joined (order of the joint).
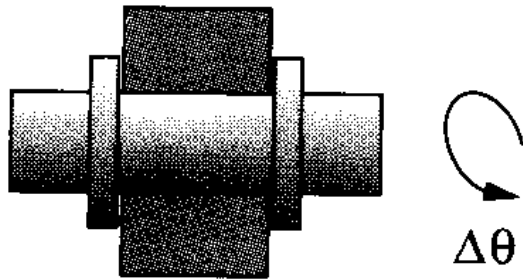
LINKS, JOINTS, AND KINEMATIC CHAINS

- **1. Classification by the Type of Contact**

  We can classify Joints by the type of contact as **Lower Pairs** or **Higher Pairs**. If joints have surface contact, they are called **Lower pair** (as with a pin surrounded by a hole). If joints have point or line contact, they are called **Higher pair**.
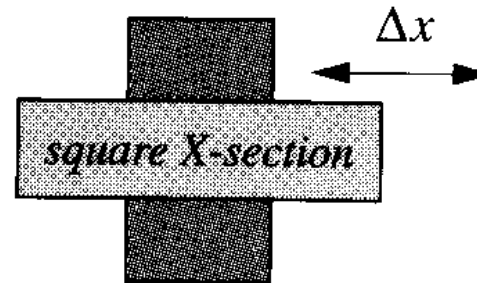
- The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces.

LINKS, JOINTS, AND KINEMATIC CHAINS

- The six possible **lower pairs** are: Revolute (R), Prismatic (P), Screw/Helical (H), Cylindric (C), Spherical (S), and Flat (F).
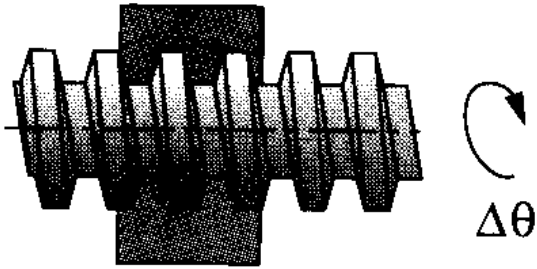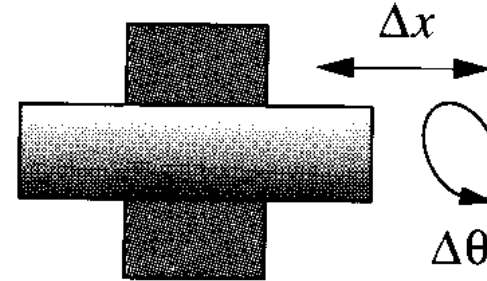
$\Delta\theta$

Revolute (R) joint—1 *DOF*

Δx

square X-section

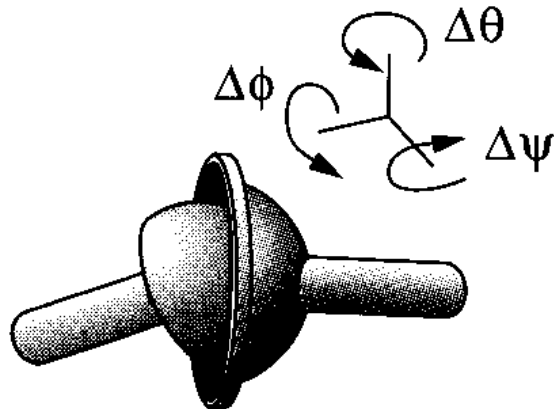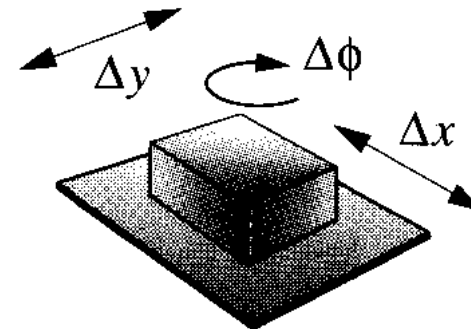Prismatic (P) joint—1 *DOF*

Helical (H) joint—1 *DOF*

Cylindric (C) joint—2 *DOF*

Spherical (S) joint—3 *DOF*

Planar (F) joint—3 *DOF*

LINKS, JOINTS, AND KINEMATIC CHAINS

- **2. Classification by the Number of Degrees of Freedom allowed at the joint**

  We can classify Joints by the number of degrees of freedom allowed at the joint as **One-Freedom** Joints or **Full Joints, Two Freedom** Joints or **Half Joints** and **Three Freedom Joints.**

- Examples of one freedom joints are : a rotating pin joint (R) and a translating slider Joint (P).

- Examples of two freedom joints are: link against plane and a pin in slot.

- Examples of three freedom joints are: a spherical, or ball-and-socket joints.
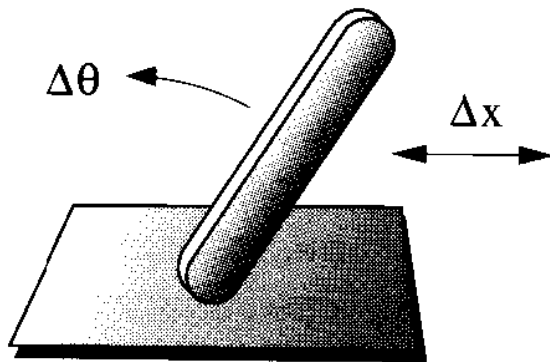
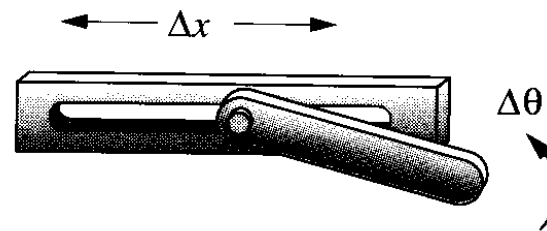# LINKS, JOINTS, AND KINEMATIC CHAINS
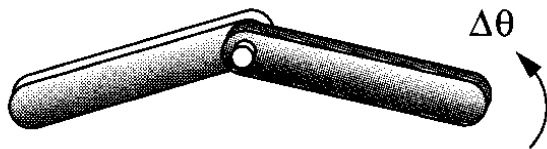
Rotating full pin (R) joint
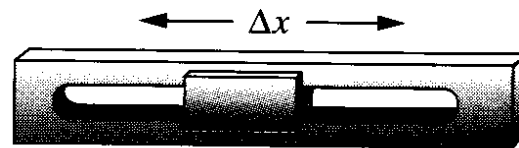
Translating full slider (P) joint

Full joints - 1 *DOF*

Link against plane

Pin in slot

Roll-slide (half or RP) joints - 2 *DOF*

LINKS, JOINTS, AND KINEMATIC CHAINS

- **3. Classification by the Type of Physical Closure of the Joint**

  We can classify Joints by the type of physical closure of the joint as **Force** closed or **Form** closed.

- A **form-closed** joint is kept together or *closed by its geometry. A pin in a hole or a slider in* a two-sided slot are form closed. In contrast, a **force-closed** joint, such as a pin in a half-bearing or a slider on a surface, *requires some external force to keep it together or closed. This force could be supplied by gravity, a spring, or any external means.*

- **4. Classification by the Order of Joints**

  We can classify Joints by the order of joints as **1st order**, **2nd order** and so on.

- *Order is defined as the number of links joined minus one.*

- It takes two links to make a single joint; thus the simplest joint combination of two links has order one.

- Joint order has significance in the proper determination of overall degree of freedom for the assembly.

LINKS, JOINTS, AND KINEMATIC CHAINS

L_1 L_2

ref. $\Delta\theta_2$
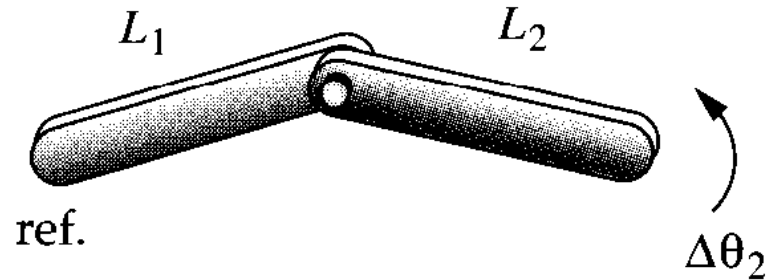
First order pin joint - one *DOF*
(two links joined)

$\Delta\theta_3$ L_3

L_2

ref. $\Delta\theta_2$

L_1
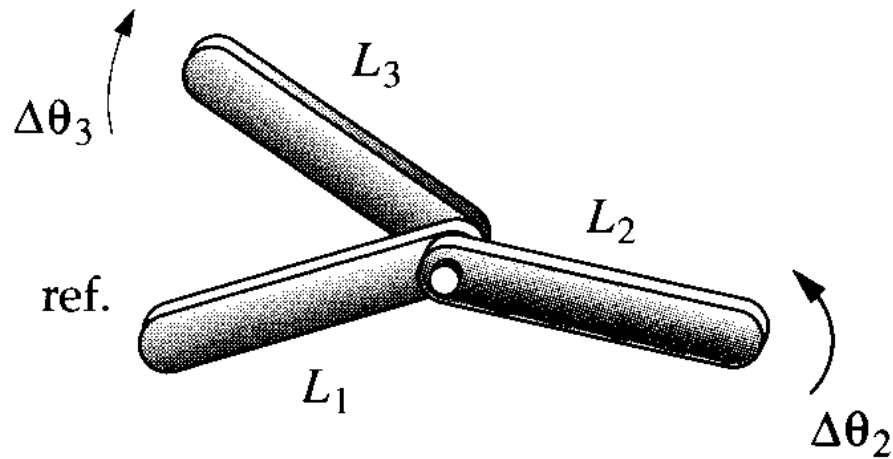
Second order pin joint - two *DOF*
(three links joined)

LINKS, JOINTS, AND KINEMATIC CHAINS

- **Kinematic Chain**

- A **kinematic chain** is defined as:

  An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

- A **mechanism** is defined as:

  A kinematic chain in which at least one link has been "grounded," or attached, to the frame of reference (which itself may be in motion).

- A **machine** is defined as:

  A combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.
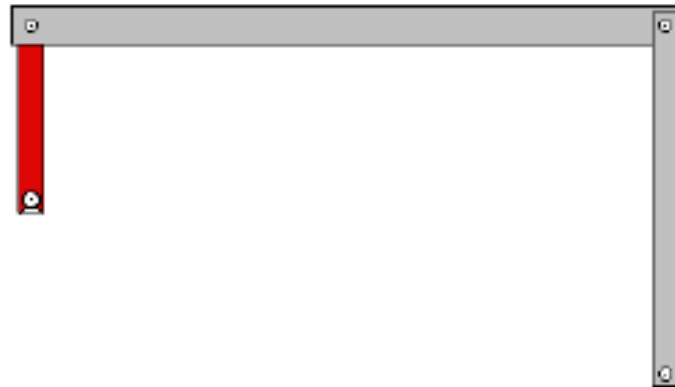
LINKS, JOINTS, AND KINEMATIC CHAINS

⦿ **Ground**

Any link or links that are fixed with respect to the reference frame

⦿ **Crank**

A link which makes a complete revolution and is pivoted to ground

- **Rocker**

  A link which has oscillatory (back and forth) rotation and pivoted to ground
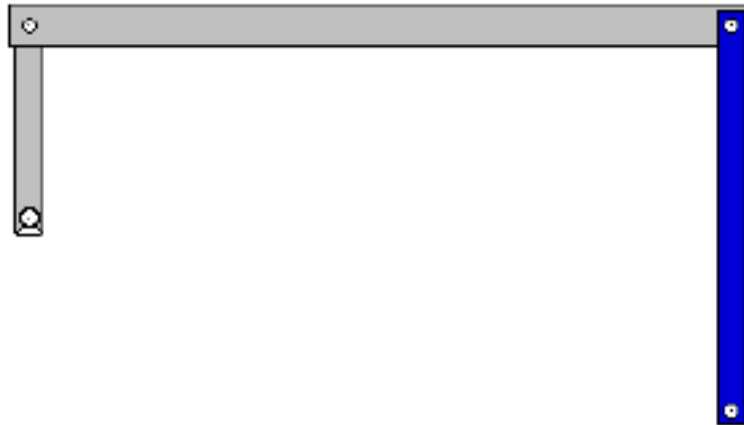
- **Coupler**

  A link which has complex motion and is pivoted to ground

DETERMINING DEGREE OF FREEDOM

- **Degree of freedom** (also called the **mobility M)** *of a system can be defined as:*

  the number of inputs which need to be provided in order to create a predictable output or the number of independent coordinates required to define its position.

- **Degree of Freedom in Planar Mechanisms**

  To determine the overall *DOF of any mechanism, we must account for the number of* links and joints, and for the interactions among them.

- **Gruebler Condition**

- Any link in a plane has 3 *DOF. Therefore, a system of L unconnected links in the same plane will have 3L DOF.* When these links are connected by a **full joint** t*his removes two DOF, and when these links are connected to* **half joint** it removes only one *DOF from the system (because a half joint* has two *DOF). In addition, when any link is* **grounded** *or attached to the reference frame, all* three of its *DOF will be removed.*

(a) Two unconnected links
DOF = 6

Δy_1 Δy_2

Δx_1 Δx_2

$\Delta\theta_1$ $\Delta\theta_2$

(b) Connected by a full joint
DOF = 4

Δy

Δx

$\Delta\theta_1$ $\Delta\theta_2$

(c) Connected by a roll-slide (half) joint
DOF = 5

$\Delta\theta_2$ Δx_2

Δy

Δx_1

$\Delta\theta_1$

23
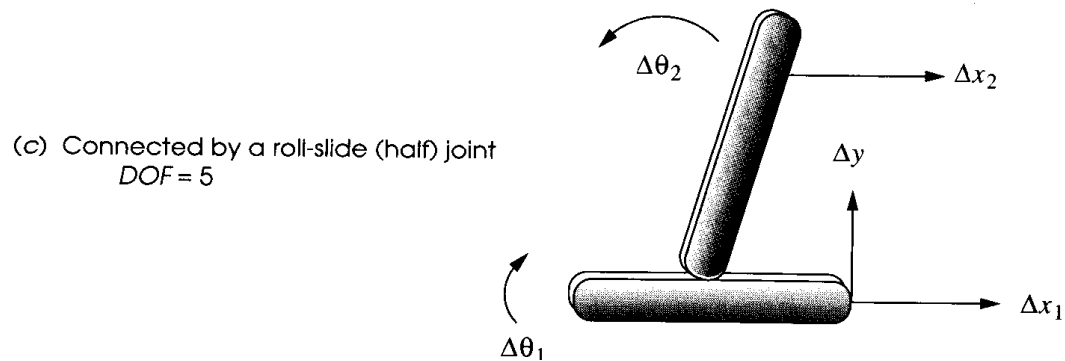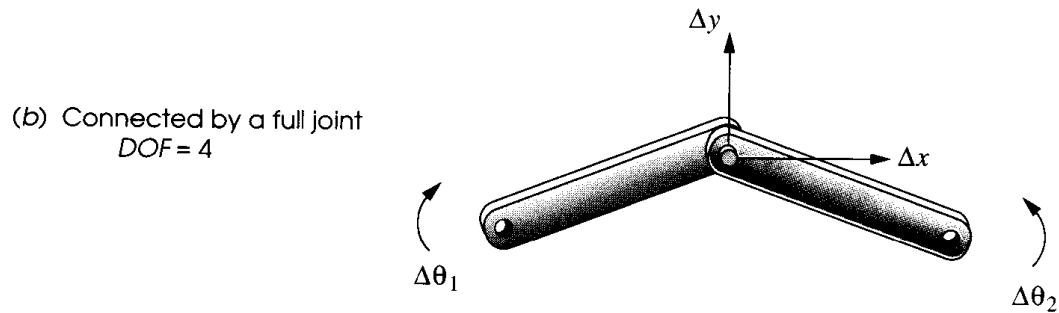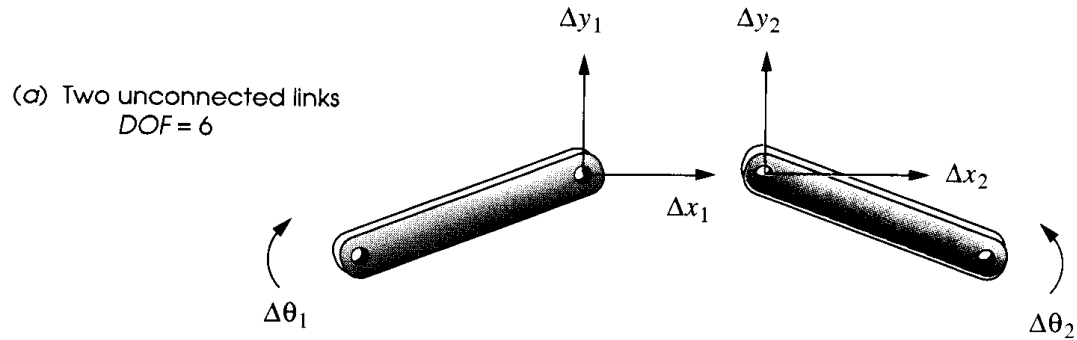
- **Gruebler's Equation**
- Based on the above reasoning, **Gruebler's Equation** is:

$$M=3L-2J-3G \qquad (2.1a)$$

where: M = *degree of freedom or mobility*
L = *number of links*
J = *number of joints*
G = *number of grounded links*

- Note that in any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger, higher-order ground link, as there can be only one ground plane. Thus G is always one, and Gruebler's equation becomes:

$$M=3(L-1)-2J \qquad (2.1b)$$

DETERMINING DEGREE OF FREEDOM

- **Kutzbach's Modification of Gruebler's Equation**
- Kutzbach's Modification of Gruebler's Equation is:

$$M=3(L-1)-2J_1-J_2 \qquad (2.1c)$$

where: M = degree of freedom or mobility
           L = number of links
           J_1 = number of 1 DOF (full) joints
           J_2 = number of 2 DOF (half) joints

- The value of J_1 and J_2 in these equations must still be carefully determined to account for all full, half, and multiple joints in any linkage.
- **Multiple joints** count as one less than the number of links joined at that joint and add to the "full" (J_1) category.

EXERCISE

- Compute the *DOF of the following examples with Kutzbach's equation.*

Ground

Multiple joint

6

4

5

2

Half joint

Ground

Ground (link 1)
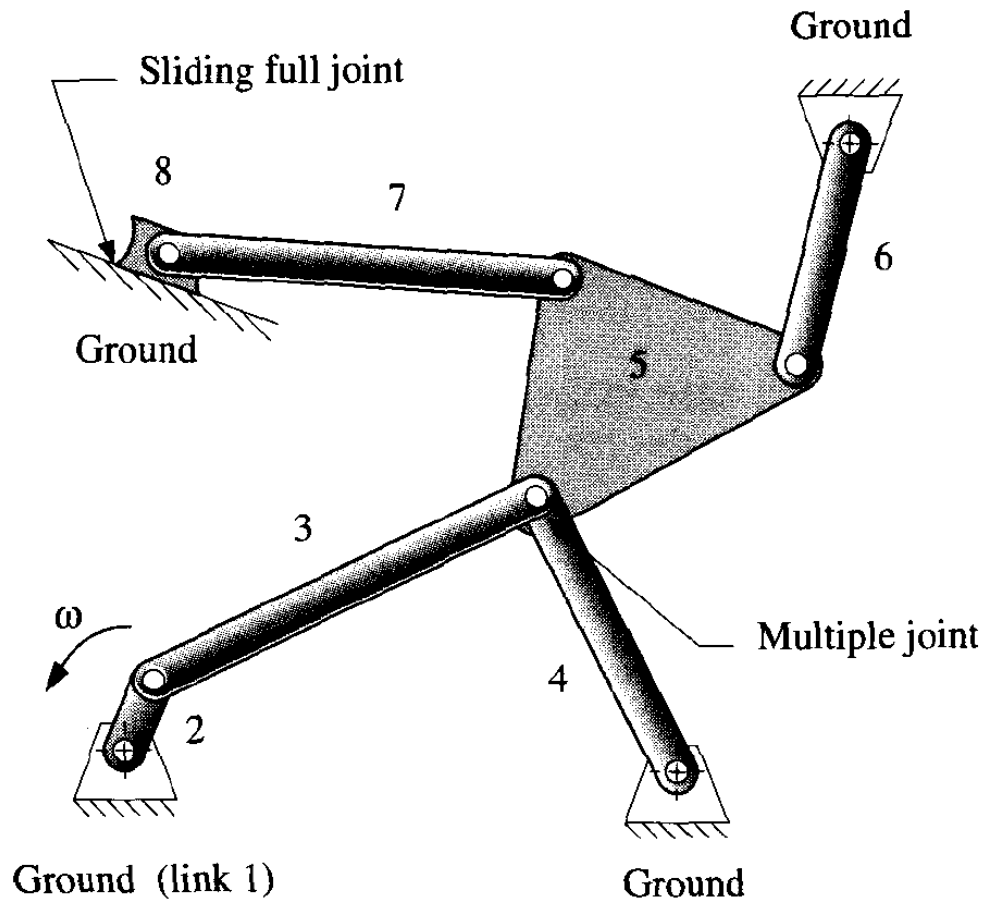
MECHANISMS AND STRUCTURES

- The degree of freedom of an assembly of links completely predicts its character. There are only three possibilities. *If the DOF is positive, it will be a **mechanism*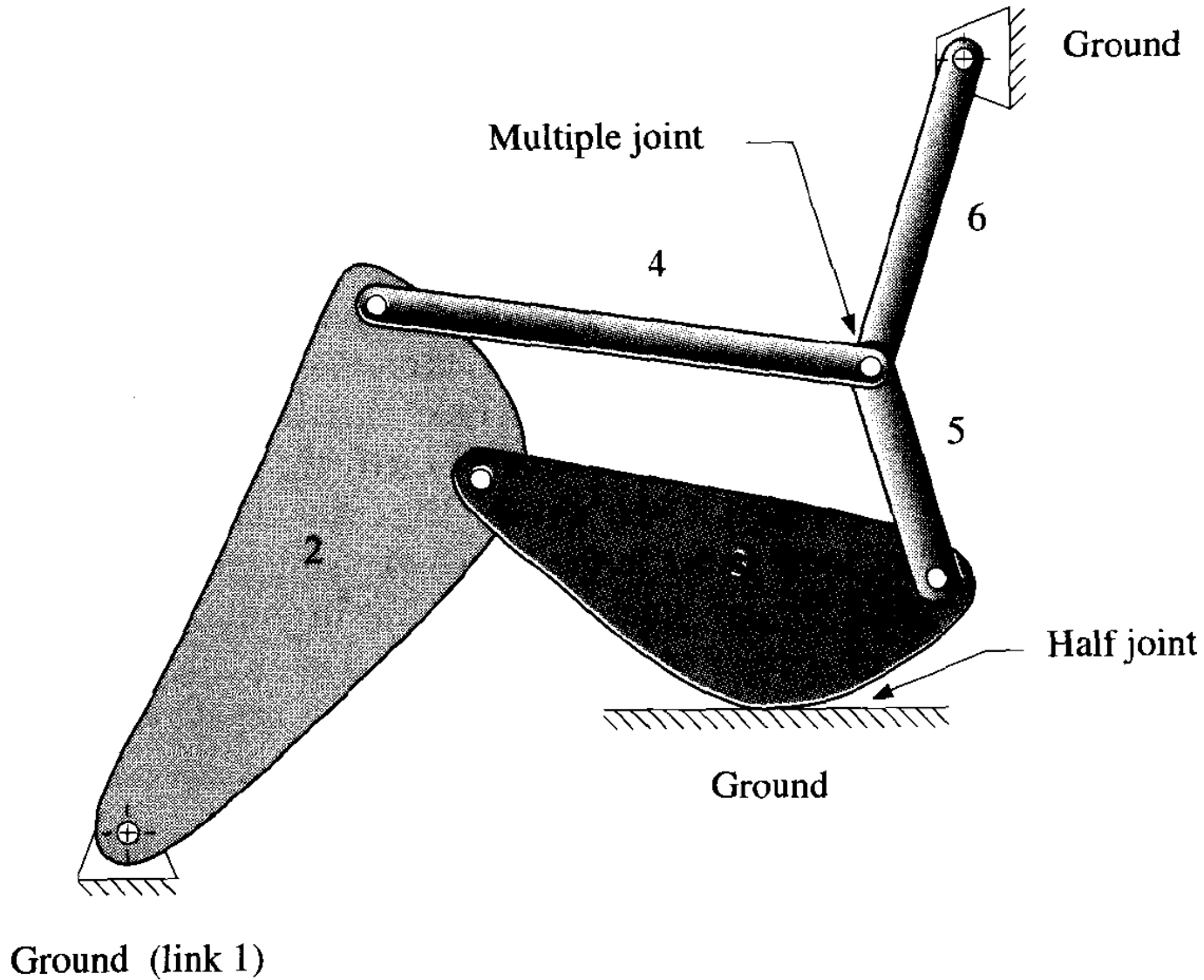*, and the links* will have relative motion. *If the DOF is exactly zero, then it will be a **structure**, and no* motion is possible. *If the DOF is negative, then it is a **preloaded structure**, which means* that no motion is possible and some stresses may also be present at the time of assembly.

MECHANISMS AND STRUCTURES
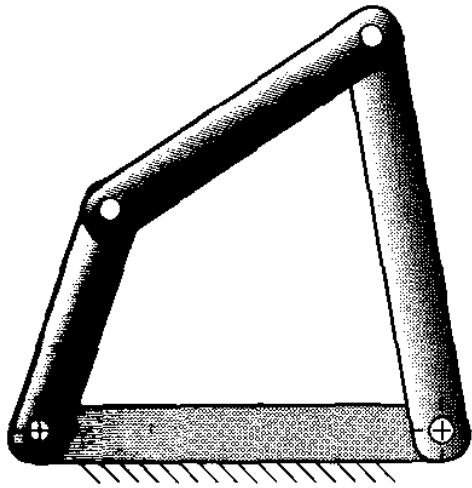
(a) Mechanism—DOF = + 1
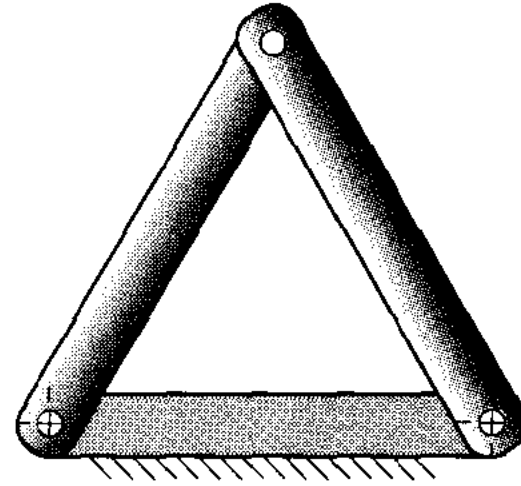
(b) Structure—DOF = 0
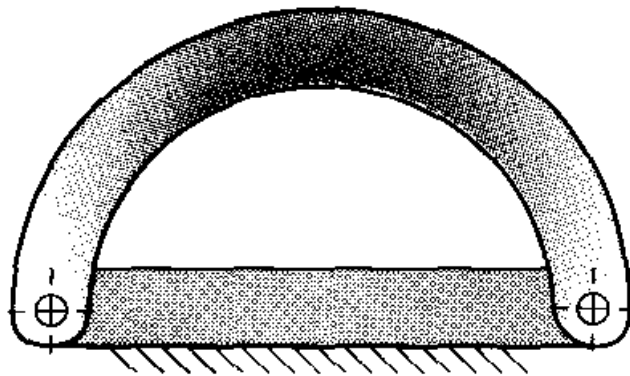
(c) Preloaded structure—DOF = −1

NUMBER SYNTHESIS

- *The determination of the number and order of **links** and **joints** necessary to produce motion of a particular DOF. **Order** in this context refers to the number of nodes per link, i.e., **binary**, **ternary**, **quaternary**, etc.*

- ***Hypothesis***: *If all joints are full joints, an odd number of DOF requires an even number of links and vice versa.*

- ***Proof***: *Given: All even integers can be denoted by 2m or by 2n, and all odd integers can be denoted by 2m - 1 or by 2n - 1, where n and m are any positive integers. The number of joints must be a positive integer.*

NUMBER SYNTHESIS

- **Let:** *L = number of links, J = number of joints, and M = DOF = 2m (i.e., all even numbers)*
- **Then:** rewriting **Gruebler's equation** (2.1b) to solve for *J*,

$$J = 3/2(L-1) - M/2$$

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2}$$

This cannot result in J being a positive integer as required.

Try: $M = 2m - 1$ and $L = 2n - 1$ (i.e., both any odd numbers):

$$J = 3n - m - \frac{7}{2}$$

This also cannot result in J being a positive integer as required.

NUMBER SYNTHESIS

Try: $M = 2m - 1$, and $L = 2n$ (i.e., odd-even):

$$J = 3n - m - 2$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Try: $M = 2m$ and $L = 2n - 1$ (i.e., even-odd):

$$J = 3n - m - 3$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

So, for our example of one-*DOF* mechanisms, we can only consider combinations of 2, 4, 6, 8 . . . links. Letting the order of the links be represented by:

B = *number of binary links*
T = *number of ternary links*
Q = *number of quaternaries*
P = *number of pentagonals*
H = *number of hexagonals*

NUMBER SYNTHESIS

the total number of links in any mechanism will be:

$$L = B + T + Q + P + H + \cdots \qquad (2.3a)$$

Since *two link nodes* are needed to make *one joint*:

$$J = \frac{nodes}{2} \qquad (2.3b)$$

and

$$nodes = order\ of\ link \times no.\ of\ links\ of\ that\ order \qquad (2.3c)$$

then

$$J = \frac{(2B + 3T + 4Q + 5P + 6H + \cdots)}{2} \qquad (2.3d)$$

Substitute Equations 2.2a and 2.3d into Gruebler's equation (2.1b)

$$M = 3(B + T + Q + P + H - 1) - 2\left(\frac{2B + 3T + 4Q + 5P + 6H}{2}\right)$$

$$(2.4)$$

$$M = B - Q - 2P - 3H - 3$$

NUMBER SYNTHESIS

In order to determine all possible combinations of links for a particular *DOF*, we must combine equations 2.1b and 2.3d:

$$\frac{3}{2}(L-1) - \frac{M}{2} = \frac{(2B+3T+4Q+5P+6H)}{2}$$

(2.5)

$$3L - 3 - M = 2B + 3T + 4Q + 5P + 6H$$

Now combine equation 2.5 with equation 2.3a to eliminate *B*:

$$L - 3 - M = T + 2Q + 3P + 4H$$ (2.6)

We will now solve equations 2.3a and 2.6 simultaneously (by progressive substitution) to determine all compatible combinations of links for *DOF* = 1, up to eight links. The strategy will be to start with the smallest number of links, and the highest-order link possible with that number, eliminating impossible combinations.

(Note: *L must be even for odd DOF.*)

NUMBER SYNTHESIS

Case 1. $L = 2$

$$L - 4 = T + 2Q + 3P + 4H = -2 \qquad (2.7a)$$

This requires a negative number of links, so $L = 2$ is impossible.

Case 2. $L = 4$

$$L - 4 = T + 2Q + 3P + 4H = 0; \qquad \text{so}: \ T = Q = P = H = 0$$

$$(2.7b)$$

$$L = B + 0 = 4; \qquad B = 4$$

The simplest one-*DOF* linkage is four binary links—the **fourbar linkage**.

Case 3. $L = 6$

$$L - 4 = T + 2Q + 3P + 4H = 2; \qquad \text{so}: \quad P = H = 0 \qquad (2.7c)$$

T may only be 0, 1, or 2; Q may only be 0 or 1

If $Q = 0$ then T must be 2 and:

$$L = B + 2T + 0Q = 6; \qquad B = 4, \qquad T = 2 \qquad (2.7d)$$

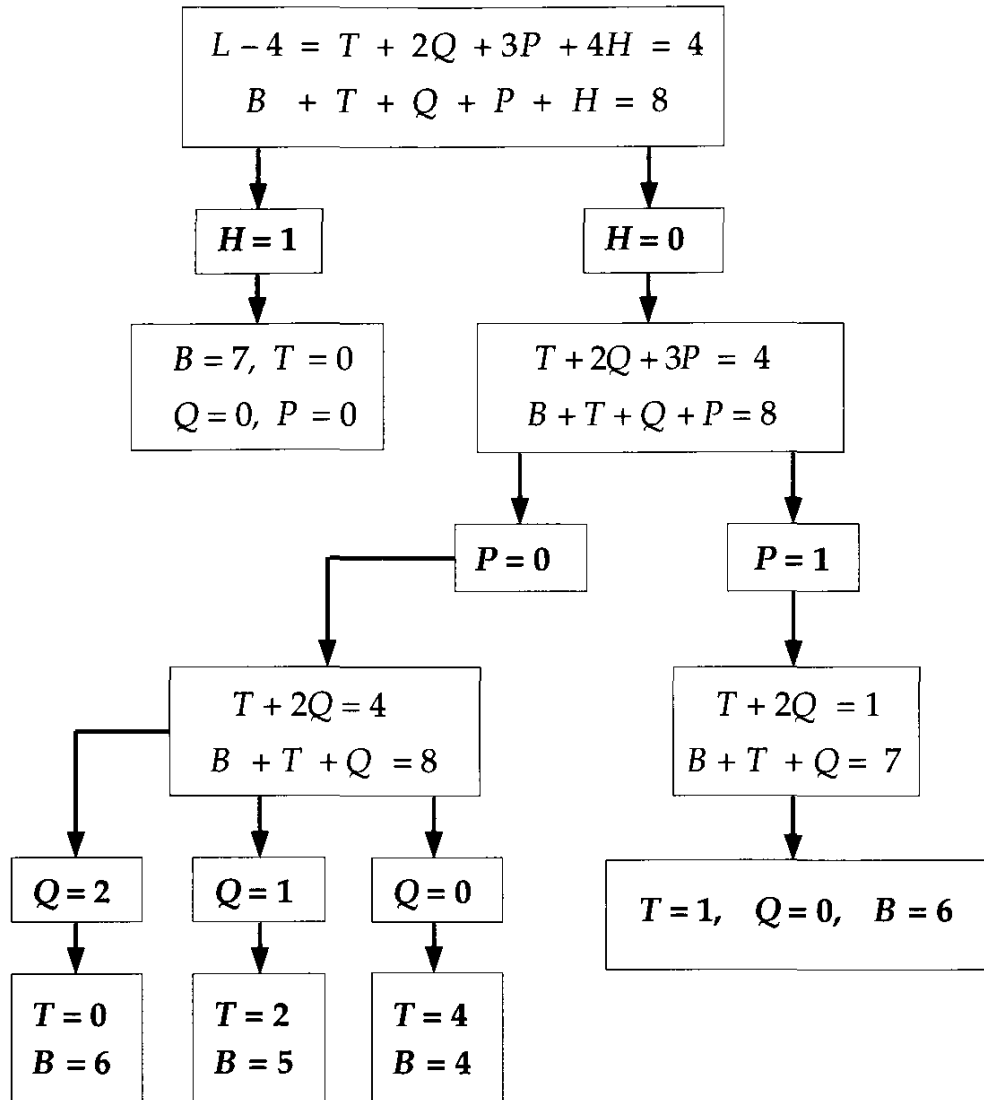If $Q = 1$, then T must be 0 and:

$$L = B + 0T + 1Q = 6; \qquad B = 5, \qquad Q = 1 \qquad (2.7e)$$

There are then two possibilities for $L = 6$. Note that one of them is in fact the simpler fourbar with two ternaries added as was predicted above.

NUMBER SYNTHESIS

CASE 4. $L = 8$

A tabular approach is needed with this many links:

$$L - 4 = T + 2Q + 3P + 4H = 4$$
$$B + T + Q + P + H = 8$$

$H = 1$

$H = 0$

$B = 7,\ T = 0$
$Q = 0,\ P = 0$

$T + 2Q + 3P = 4$
$B + T + Q + P = 8$

$P = 0$

$P = 1$

$T + 2Q = 4$
$B + T + Q = 8$

$T + 2Q = 1$
$B + T + Q = 7$

$Q = 2$

$Q = 1$

$Q = 0$

$T = 1,\ Q = 0,\ B = 6$

$T = 0$
$B = 6$

$T = 2$
$B = 5$

$T = 4$
$B = 4$

(2.7f)

NUMBER SYNTHESIS

- From this analysis we can see that, for one DOF, there is only one possible four link configuration, two six link configurations and five possibilities for eight links using binary through hexagonal links.

NUMBER SYNTHESIS

TABLE 2-2   1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Binary	Link Sets			
		Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0